RAMAKRISHNA MISSION VIDYAMANDIRA

(A Residential Autonomous College)

Belur Math, Howrah

B.A./B.Sc. 1st Semester (July – December 2010)

Mid-Semester Examination, September 2010

Date: 06.09.2010 **Mathematics (Honours)** Full Marks 50

Time: 11 am – 1 pm

Group - A

Answer any **two**: 2x5

1. a) Find *sup A* and *inf A* where $A = \left\{ \frac{n+(-1)^n}{n} : n \in \mathbb{N} \right\}$. 2

   b) Let S and T be two non-empty bounded subsets of $\mathbb{R}$ and $U = \{x+y : x \in S, y \in T\}$. Prove that *sup U* = *sup S* + *sup T*. 3

2. a) State and prove well-ordering principle of $\mathbb{N}$. 1+2

   b) Prove that for all $n \in \mathbb{N}$, $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is an even integer. 2

3. a) Prove that arbitrary union of open sets in $\mathbb{R}$ is an open set. Is the result true for arbitrary intersection? Justify. 2+1

   b) Give an example of an infinite set $S \subseteq \mathbb{R}$ such that (i) S has no limit point, (ii) S has only one limit point. 2

Group - B

Answer any **three** questions: 3x5

4. a) Let $f : X \to Y$ be an injective map and $A \subseteq X$. Prove that $A = f^{-1}(f(A))$. 3

   b) Define a bijective map from $\mathbb{Q}$ onto $\mathbb{Q} \cup \{\sqrt{2}\}$. 2

5. a) Find the total number of reflexive relations on a set containing *n* elements. 3

   b) Consider the relation ρ defined on $\mathbb{Z}$ by "$a \ \rho \ b$ iff $[a-b] = 0 \ (a,b \in \mathbb{Z})$". Verify whether ρ is transitive. 2

6. a) Let ρ be an equivalence relation on a set S and $a, b \in S$. Prove that $[a] = [b]$ iff $a \rho b$. 3

   b) Define an equivalence relation on $\{1, 2, 3\}$. 2

7. If H, K are two subgroups of a group G then show that HK is a subgroup of G iff $HK = KH$ where $HK = \{xy : x \in H, y \in K\}$ and KH has a similar definition. 5

8. a) Define $Z(G)$, the centre of a group G. Prove that $Z(G)$ is a subgroup of G. 1+3

   b) Prove that G is abelian if $G = Z(G)$. 1

Group - C

9. Answer any **three**: 3x5

   a) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating factor of the differential equation: $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ and hence solve it. 5

   b) Find the general and singular solution of the non-linear differential equation: $(px - y)(x - py) = 2p$ where $p \equiv \frac{dy}{dx}$. 5

   c) Solve: $(y^2 + 2x^2 y)dx + (2x^3 - xy)dy = 0$. 5

d) Solve: $y = 3px + 6p^2y^2$ where $p \equiv \frac{dy}{dx}$. 5

e) Show that the general solution of the equation $\frac{dy}{dx} + Py = Q$ can be written in the form $y = K(u-v)+v$, where K is a constant and u, v are two particular solutions of it. 5

Group - D

Answer any **two**: 2x5

10. If the position vectors of four points relative to the origin O are $-6\hat{i}+3\hat{j}+2\hat{k}$, $3\hat{i}-2\hat{j}+4\hat{k}$, $5\hat{i}+7\hat{j}+3\hat{k}$ and $-13\hat{i}+17\hat{j}-\hat{k}$; show that the four points are coplanar. 5

11. Show by vector method that external bisector of any angle of a triangle divides the opposite side externally in the ratio of the lengths of two sides containing the angle. 5

12. i) Prove that $\vec{a}\times\vec{b} = [(\hat{i}\times\vec{a})\cdot\vec{b}]\hat{i} + [(\hat{j}\times\vec{a})\cdot\vec{b}]\hat{j} + [(\hat{k}\times\vec{a})\cdot\vec{b}]\hat{k}$. 3

    ii) Prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are coploanar if and only if $\vec{a},\vec{b},\vec{c}$ are coplanar. 2
